

Show work for full credit. Circle, box, or highlight your answers when answer lines are not given.

Each question's point value appears in < >.

Divide using long division. Write any remainder as a fraction of the divisor: <5 ea.>

1. $(x^3 + 5x^2 + 12x + 6) \div (x - 2)$

$$\begin{array}{r}
 x^2 + 7x + 26 + \frac{58}{x-2} \\
 x-2 \overline{) \begin{array}{l} x^3 + 5x^2 + 12x + 6 \\ -x^3 + 2x^2 \\ \hline 7x^2 + 12x \\ -7x^2 + 14x \\ \hline 26x + 6 \\ -26x + 52 \\ \hline 58 \end{array} }
 \end{array}$$

$$x^2 + 7x + 26 + \frac{58}{x-2}$$

Divide using synthetic division. Write any remainder as a fraction of the divisor: <3 ea.>

2. $(7x^3 + 9x^2 - 42) \div (x - 3)$

$$\begin{array}{r|rrrr}
 3 & 7 & 9 & 0 & -42 \\
 & & 21 & 90 & 270 \\
 \hline
 & 7 & 30 & 90 & 228
 \end{array}$$

$$7x^2 + 30x + 90 + \frac{228}{x-3}$$

-more-

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List the possible rational roots of $P(x)$ given by the Rational Root (Locator) Theorem: <5 ea.>

3. $P(x) = 4x^3 - 2x^2 - 5x + 3$

$$\frac{3}{4} : \frac{1, 3}{1, 2, 4} \Rightarrow$$

$$\pm \left\{ 1, \frac{1}{2}, \frac{1}{4}, 3, \frac{3}{2}, \frac{3}{4} \right\}$$

Write a polynomial function with the given roots: <5 ea.>

4. $\left. \begin{matrix} 2 \text{ and } 8i \\ -8i \end{matrix} \right\} \begin{matrix} +1 & 0 \\ x & : 64 \end{matrix} \left\{ \begin{matrix} (x^2 + 64) \\ (x-2) \end{matrix} \right\}$

$$y = (x-2)(x^2 + 64)$$

Use Descartes' Rule of Signs to determine the possible number of positive and negative real zeros: <5 ea.>

5. $P(x) = 9x^3 + 5x^2 + 1x - 3$

pos: + + + -

neg: - + - -

1 pos real root
2 or 0 neg. real roots

-more-

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Find all the zeros for each function: <5 ea.>

6. $P(x) = 2x^4 + 3x^3 - 17x^2 - 27x - 9$

(hint: try -1)

$$\begin{array}{r|rrrrr} -1 & 2 & 3 & -17 & -27 & -9 \\ & & -2 & -1 & 18 & 9 \\ \hline & 2x^3 & +1x^2 & -18x & -9 & 0 \end{array}$$

$$x^2(2x+1) - 9(2x+1) = 0$$

$$(2x+1)(x^2-9) = 0$$

$$(2x+1)(x+3)(x-3) = 0$$

$$x = -1 \quad x = -1/2 \quad x = \pm 3$$

$$x = \{-1, 3, -3, -1/2\}$$

7. $P(x) = 2x^3 - 3x^2 - 18x - 8$

1: $2 - 3 - 18 - 8$ NO

-1: $-2 - 3 + 18 - 8$ NO

2: $16 - 12 - 36 - 8$ NO

-2: $-16 - 12 + 36 - 8$ YES

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -18 & -8 \\ & & -4 & 14 & 8 \\ \hline & 2 & -7 & -4 & 0 \end{array}$$

$$2x^2 - 7x - 4 = 0$$

$$x^2 - 7x - 8 = 0$$

$$(x - \frac{8}{2})(x + \frac{1}{2}) = 0$$

$$(x - 4)(2x + 1) = 0$$

$$x = 4 \quad x = -1/2$$

$$x = \{-2, 4, -1/2\}$$

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-more-

Factor each polynomial completely: <5 ea.>

8. $3x^3 + 7x^2 - 20x$

$$x(3x^2 + 7x - 20)$$

$$x(x^2 + 7x - 60)$$

$$x(x + \frac{12}{3})(x - \frac{5}{3})$$

$$x(x+4)(3x-5)$$

$$\begin{array}{r} 60 \\ \wedge \\ 6 \quad 10 \\ 15 \quad 4 \\ \wedge \quad \wedge \\ 3 \quad 5 \quad 2 \quad 2 \\ \hline 12 \quad 5 \quad \checkmark \end{array}$$

9. $8x^3 - 1$

$$(2x-1)(4x^2+4x+1)$$

10. $x^4 - 3x^2 - 40$

$$(x^2 - 8x + 5)$$