Types of Conic sections

1) Simplify by combining like terms, then use the chart to determine the type of conic section.

2) chart: NOTE: Shaded box mean that it doesn't matter :)

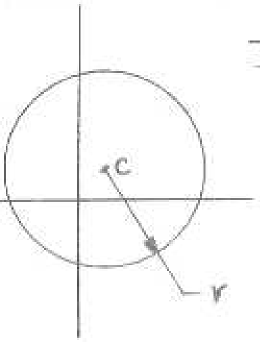
	Number of squared variables	coefficients of x^2, y^2	separated by a + or − sign
Circle	2	same	+
Ellipse	2	different	+
Hyperbola	2		—
Parabola	1		
Rectangular Hyperbola	none		

3) The "Hand Rule":

Use one or two hands to represent the shape of the conic.

A) One hand is the parabola, two hands are the circle, ellipse, and hyperbola; the number of hands used is the same as the number of squared variables.

B) If the hands are touching at the finger tips to make the shape, then the terms are joined by a + sign; if the hands are separated, then the terms are separated by a − sign.



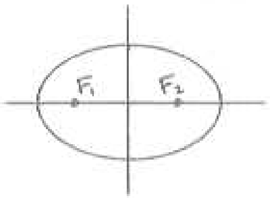
The circle

$$(x-h)^2 + (y-k)^2 = r^2$$

C is the center ; $C(h, k)$

r is the radius

The ellipse



$a > b$

foci @ $\{(\pm c, 0)\}$

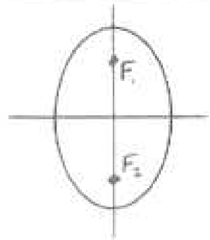
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

F_1 & F_2 are the foci

$c^2 = |a^2 - b^2|$

$x\text{-int} = \{(\pm a, 0)\}$

$y\text{-int} = \{(0, \pm b)\}$

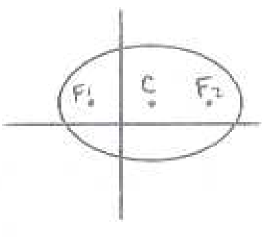


$a < b$

foci @ $\{(0, \pm c)\}$

Horizontal axis $\longleftrightarrow$ 2a Vertical axis $\updownarrow$ 2b

major axis is longer ; minor axis is shorter



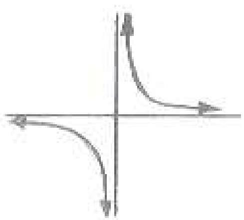
If ellipse is not "centered" on the origin, then equation becomes :

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where C represents the "center" of the ellipse ; $C(h, k)$

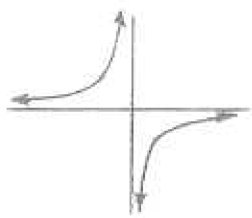
The Rectangular Hyperbola

$$xy = k$$



k is positive

Verticies @ $\{(\sqrt{k}, \sqrt{k}), (-\sqrt{k}, -\sqrt{k})\}$



k is negative

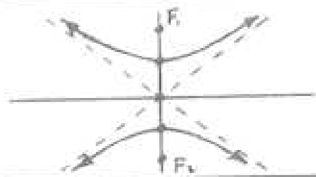
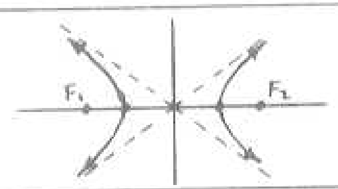
verticies @

$\{(-\sqrt{|k|}, \sqrt{|k|}), (\sqrt{|k|}, -\sqrt{|k|}$

The Parabola

Shape	 or 	 or 
Equation	$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$
axis of symm.	$x = h$	$y = k$
Vertex	(h, k)	(h, k)
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$

The Hyperbola

Shape		
Equation	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Asymptotes	$y = \pm \frac{b}{a} x$	$y = \pm \frac{b}{a} x$
Foci	$(0, \pm c)$	$(\pm c, 0)$
Verticies	$(0, \pm b)$	$(\pm a, 0)$

★ distance from center to foci $\Rightarrow c = \sqrt{a^2 + b^2}$

★ dotted lines are asymptotes; curve gets forever closer to these lines, but never touch them.